

Midterm Introduction to Logic

A1.a) Key: D: The doctor prescribes vitamin D
 U: you are under three (years old)
 O: you are over 80 years old
 $D \rightarrow (U \vee O)$

b) key: K: it keeps raining
 A: she is angry
 F: she is afraid
 $K \wedge \neg A \wedge \neg F$, Also correct:
 $K \wedge \neg (A \vee F)$

c) key: W: the midterm is on wednesday
 F: the midterm is on Friday
 L: the lecture is on wednesday
 R: the lecture is on Friday
 $\neg((W \vee F) \wedge (L \vee R))$
 (The sentence $\neg((W \wedge L) \vee (F \wedge R))$ corresponds to "it is

A2. Translation key: not the case that the midterm and the lecture were both on Wednesday or both on Friday"
 a: Anand
 b: Bhashar
 h: Harry Potter
 d: Deepika
 $K(x,y)$: x knows y
 $L(x,y)$: x loves y
 $H(x)$: x is happy
 $P(x,y,z)$: x likes y more than x likes z

a) $(K(a,b) \wedge K(b,a)) \rightarrow L(b,h)$
 b) $\neg(L(b,a) \vee L(b,d)) \wedge H(b)$. Also correct:
 $\neg L(b,a) \wedge \neg L(b,d) \wedge H(b)$
 c) $P(a,h,a) \vee P(b,a,h)$. Also correct: $\neg P(b,a,h) \rightarrow P(a,h,a)$
 [Alternative with a four place $M(x,y,z,w)$: x likes y more than z likes w.
 $M(a,h,a,a) \vee M(b,a,b,h)$]

Midterm 9 Dec 2015, contd.

3a	1	$P \rightarrow Q$	
	2	$R \rightarrow S$	
	3	$P \vee R$	
	4	P	
	5	Q	\rightarrow Elim: 1, 4
	6	$S \vee Q$	\vee Intro: 5
	7	R	
	8	S	\rightarrow Elim: 2, 7
	9	$S \vee Q$	\vee Intro: 8
	10	$S \vee Q$	\vee Elim: 3, 4-6, 7-9
	11	$(P \vee R) \rightarrow (S \vee Q)$	\rightarrow Intro: 3-10

3b.	1	$\neg P(a) \rightarrow P(b)$	
	2	$\neg P(b)$	
	3	$b = a$	
	4	$\neg P(a)$	$=$ Elim: 2, 3
	5	$P(b)$	\rightarrow Elim: 1, 4
	6	\perp	\perp Intro: 5, 2
	7	$\neg(b = a)$	\neg Intro: 3-6

3c)	1 $\neg Q \vee R$	
	2 $\neg Q$	
	3 $Q \wedge P \wedge \neg R$	
	4 Q	\wedge Elim: 3
	5 \perp	\perp Intro: 4, 2
	6 $\neg(Q \wedge P \wedge \neg R)$	\neg Intro: 3-5
	7 R	
	8 $Q \wedge P \wedge \neg R$	
	9 $\neg R$	\wedge Elim: 8
	10 \perp	\perp Intro: 7, 9
	11 $\neg(Q \wedge P \wedge \neg R)$	\neg Intro: 8-10
	12 $\neg(Q \wedge P \wedge \neg R)$	\vee Elim: 1, 2-6, 7-11

or in 10 lines:

	1 $\neg Q \vee R$	
	2 $Q \wedge P \wedge \neg R$	
	3 Q	\wedge Elim: 2
	4 $\neg R$	\wedge Elim: 2
	5 $\neg Q$	
	6 \perp	\perp Intro: 3, 5
	7 R	
	8 \perp	\perp Intro: 7, 4
	9 \perp	\vee Elim: 1, 5-6, 7-8
	10 $\neg(Q \wedge P \wedge \neg R)$	\neg Intro: 2-9

3d.

1	$P \leftrightarrow \neg Q$	
2	$Q \wedge R$	
3	Q	\wedge Elim: 2
4	R	\wedge Elim: 2
5	P	
6	$\neg Q$	\leftrightarrow Elim: 1, 5
7	\perp	\perp Intro: 3, 6
8	$\neg P$	\neg Intro: 5-7
9	$\neg P \wedge R$	\wedge Intro: 8, 4
10	$\neg P \wedge R$	
11	$\neg P$	\wedge Elim: 10
12	R	\wedge Elim: 10
13	$\neg Q$	
14	P	\leftrightarrow Elim: 1, 13
15	\perp	\perp Intro: 14, 11
16	$\neg \neg Q$	\neg Intro: 13-15
17	Q	\neg Elim: 16
18	$Q \wedge R$	\wedge Intro: 17, 12
19	$(Q \wedge R) \leftrightarrow (\neg P \wedge R)$	\leftrightarrow Intro: 2-9, 10-18

Bonus

1.	$\neg A \rightarrow (B \wedge C)$	
2.	$\neg A \rightarrow (\neg B \vee \neg C)$	
3.	$\neg A$	
4.	$B \wedge C$	\rightarrow Elim: 1, 3
5	B	\wedge Elim: 4
6	C	\wedge Elim: 4
7.	$\neg B \vee \neg C$	\rightarrow Elim: 2, 3
8	$\neg B$	
9	\perp	\perp Intro: 5, 8
10	$\neg C$	
11	\perp	\perp Intro: 6, 10
12	\perp	\vee Elim: 7, 8-9, 10-11
13	$\neg \neg A$	\neg Intro: 3-12
14	A	\neg Elim: 13

B1 a) $(P \leftrightarrow \neg Q) \leftrightarrow (\neg R \rightarrow P)$ $\stackrel{\text{def } \leftrightarrow, \text{ def } \rightarrow}{\iff}$

$((P \wedge \neg Q) \vee (\neg P \wedge \neg \neg Q)) \leftrightarrow (\neg \neg R \vee P)$ $\stackrel{\text{double neg}}{\iff}$

$((P \wedge \neg Q) \vee (\neg P \wedge Q)) \leftrightarrow (R \vee P)$ $\stackrel{\text{def } \leftrightarrow}{\iff}$

$((P \wedge \neg Q) \vee (\neg P \wedge Q)) \wedge (R \vee P) \vee \neg((P \wedge \neg Q) \vee (\neg P \wedge Q)) \wedge \neg(R \vee P)$ $\stackrel{\text{de Morgan, 2x}}{\iff}$

$((P \wedge \neg Q) \vee (\neg P \wedge Q)) \wedge (R \vee P) \vee \neg(P \wedge \neg Q) \wedge \neg(\neg P \wedge Q) \wedge (\neg R \wedge \neg P)$ $\stackrel{\text{de Morgan, 2x}}{\iff}$
 $\hookrightarrow ((P \vee \neg \neg Q) \wedge (\neg \neg P \vee \neg Q) \wedge \neg R \wedge \neg P) \vee ((\neg P \vee \neg \neg Q) \wedge (P \vee \neg Q) \wedge \neg R \wedge \neg P)$ $\stackrel{\text{double neg.}}{\iff}$
 $\hookrightarrow ((P \wedge \neg Q) \vee (\neg P \wedge Q)) \wedge (R \vee P)$

$((P \wedge \neg Q) \vee (\neg P \wedge Q)) \wedge (R \vee P) \vee ((\neg P \vee Q) \wedge (P \vee \neg Q) \wedge \neg R \wedge \neg P)$. This is in NNF
 (Other solutions possible, e.g. if you use
 $A \leftrightarrow B \iff (\neg A \vee B) \wedge (\neg B \vee A)$ instead of
 $A \leftrightarrow B \iff (A \wedge B) \vee (\neg A \wedge \neg B)$

b) $\neg(P \vee \neg Q) \vee (R \wedge S \wedge T)$ $\stackrel{\text{de Morgan}}{\iff}$
 $(\neg P \wedge \neg \neg Q) \vee (R \wedge S \wedge T)$ $\stackrel{\text{double neg}}{\iff}$
 $(\neg P \wedge Q) \vee (R \wedge S \wedge T)$ $\stackrel{\text{distr. law}}{\iff}$
 $(\neg P \wedge Q) \vee R) \wedge ((\neg P \wedge Q) \vee S) \wedge ((\neg P \wedge Q) \vee T)$ $\stackrel{\text{distr. law}}{\iff}$
 $(\neg P \vee R) \wedge (Q \vee R) \wedge (\neg P \vee S) \wedge (Q \vee S) \wedge (\neg P \vee T) \wedge (Q \vee T)$
 this is in CNF

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|-------------|----------|
| B2 a) false | f) false |
| b) true | g) false |
| c) true | h) true |
| d) true | i) false |
| e) false | j) false |